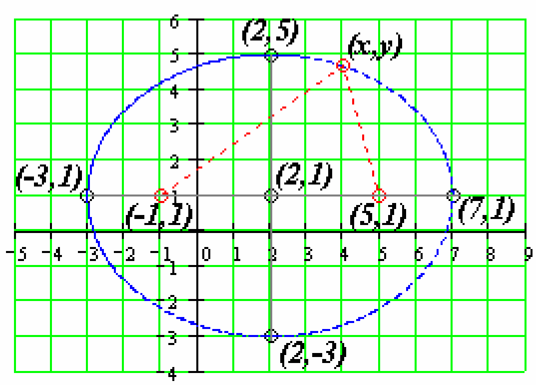
Analytical Geometry:

Generally speaking, this topic refers to circles, parabolas, ellipses and hyperbolas. It is important to know the formulas of these and what the various parts represent.

Circles: The definition of a circle is the set of all points in a plane that are equidistant from a given point, the center. The equal distance is, of course, the radius.

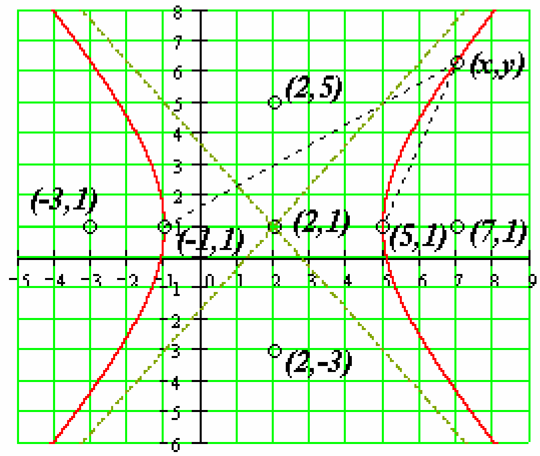
Given the center points (h,k) and radius *r*, the equation of the circle is $(x-h)^2+(y-k)^2=r^2$. If the equation is in standard form, $x^2+y^2+Bx+Cy+D=0$, you can complete the square to find the center and radius.

Ellipses: An ellipse has to foci in addition to the center. The definition is the set of all points in a plane so that the sum of the distances from the two foci is constant. While the points can be anywhere in the plane, we almost always work with foci on the same vertical or horizontal line.



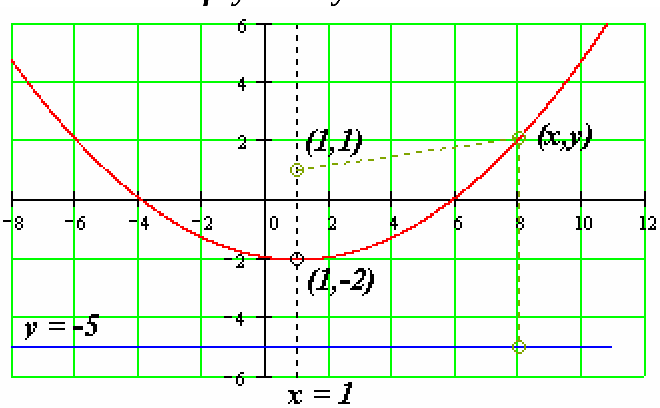
The ***center*** is (2,1) and the ***foci*** are (-1,1) and (5,1). The ***vertices*** (-3,1) and (7,1) are the endpoints of the ***major axis***. The points (2,5) and (2,-3) are endpoints of the ***minor axis***. The equation of this ellipse is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, where 2*a* is the length of the major axis, 2*b* the length of the minor axis, (*h*,*k*) the center, and the foci are *c* units from the center along the major axis, and $a^2-b^2=c^2$. Of course, if $b>a$ the major axis is vertical and $b^2-a^2=c^2$. The ***eccentricity*** of an ellipse is the ratio c/a (horizontal major axis) or c/b (vertical axis). The length of the major axis is the same as the sum of the distances from any point to the two foci. In standard form, $Ax^2+By^2+Cx+Dy+E=0$, *A* and *B* will have the same sign, but are unequal. Complete the square to get the equation in the desired form.

Hyperbolas: The definition of a hyperbola is similar to that of the ellipse, except that the absolute value of the distance from any point to the two foci is constant. We still have a center, foci, vertices, and major axis.



The center is (2,1) and the foci are (-3,1) and (7,1). The vertices (5,1) and (-1,1) are the endpoints of the transverse axis. The equation of the hyperbola is $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$, if the transverse axis is horizontal. The center is still (*h*,*k*). The foci are *c* units from the center, along the line containing the transverse axis, where $c^2=a^2+b^2$. In addition, hyperbolas have asymptotes. Their equations are $y-k=\pm\frac{b}{a}(x-h)$. If the transverse axis is vertical, the equation is $\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$. The eccentricity is $e={}^{c}\!/_{a}$ or ${}^{c}\!/_{b}$, depending on the transverse axis.

Parabolas: A parabola is the set of all points in a plane that are equidistant from a given point (focus) and a given line (directrix). The simple form requires that the directrix be either horizontal or vertical. For a horizontal directrix, the equation is $y-k=\frac{1}{4p}(x-h)^2$, where (*h*,*k*) is the vertex and *p* is the distance from the vertex to the focus. The line $x=h$ is ***the axis of symmetry***.



The standard form for this equation is $y = ax^2 + bx + c$. In this form, the vertex is $\left(-b/2a, (4ac - b^2)/4a \right)$.

If the directrix is vertical, the *x*- and *y*-terms are interchanged. The equations becomes $x - h = \frac{1}{4p}(y - k)^2$.

Problems:

1. The vertex of the graph of the parabola $y = 3x^2 + bx + c$ is at (-2,-7); determine the value of *b*-*c*; **NC ALGII 2002 33**
2. The distance between the centers of the circles $x^2 - 8x + y^2 - 28y + 112 = 0$ and $x^6 + 6x + y^2 + 20y - 12 = 0$ is: **FURMAN 2003 JR11**
3. Through which quadrants does the circle $x^2 + 4x + y^2 - 6y + 1 = 0$ pass? **FURMAN 2003 JR13**
4. If a parabola has vertex (0,0) and focus (1,0), the equation is: **FURMAN 2003 JR 15**
5. If Q is the point on the circle $x^2 - 10x + y^2 + 6y + 29 = 0$ which is furthest from the point $P(-1,-6)$, then the distance from P to Q is**: FURMAN 2003 SR 2.**
6. The parabola $y = ax^2 + bx + c$ intersects the *y*-axis in the point (0,8) and intersects the *x*-axis in the single point (2,0). How many pairs of integers (m,n) with $-2000 \le m \le 2000$ lie on the parabola? **NC SMC 2000 INT 4**
7. Let f(x) be a quadratic polynomial such that f(3) = 15 and f(-3) = -9. Find the coefficient of x in f(x). **NC SMC 2002 MC 1**
8. The center © and the vertices (V) of the ellipse $4x^2 + 9y^2 - 16x - 54y + 61 = 0$ are**: NC ALGII 2000 9**
9. The quadratic equation $y = ax^2 + bx + c$ is known to pass through the points (0,5), (2,11) and (-2,15). Find the sum of a and b**. NC ALGII 2000 17**
10. Write an equation for the hyperbola with horizontal transverse axis and asymptotes $y = \pm 4/3\, x$. **NC ALGII 2000 25**
11. A parabola of the form $y = x^2 + bx + c$ contains the points (2,3) and (4,3), Find the value of c. **AMC10 2006 .**