Algebra

Series

Arithmetic series:

a_1 = 1st term, a_n = nth term, d = common difference

nth term: $a_n = a_1 + (n-1)d$

Sum of 1st n terms: $\sum_{k=1}^{n} a_k = \frac{n}{2}\left(2a_1 + (n-1)d\right) = \frac{n}{2}\left(a_1 + a_n\right)$

Arithmetic mean of a and b: $\frac{a+b}{2}$

Geometric series:

a_1 = 1st term, a_n = nth term, r = common ratio

nth term: $a_n = ar^{n-1}$

Sum of 1st n terms: $\sum_{k=1}^{n} a_k = \frac{a_1(r^n - 1)}{r-1}$

Sum of infinite geometric sequences ($|r| < 1$): $\sum_{k=1}^{\infty} a_k \frac{a}{1-r}$

Geometric mean of a and b: $\sqrt{ab}$

Harmonic series: A sequence of numbers is harmonic if the reciprocals of the numbers form an arithmetic progression.

Harmonic mean of a and b: $\left(\frac{\frac{1}{a} + \frac{1}{b}}{2}\right)^{-1} = \frac{2ab}{a+b}$

If A, G, and H respectively represent the arithmetic, geometric, and harmonic mean between a and b, then $G^2 = AH$.

Binomial Theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$, where $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

When x and y are replaced by probabilities p and q, with $p+q=1$, the term at k represents the probability of getting k successful events with probability q and the other n-k events of probability p.

Sum of binomial coefficients: replace x and y both with 1, $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

Multinomial expansions:

For an expansion $(x_1 + x_2 + \cdots x_k)^n$, the coefficient of the term $x_1^{e_1} x_2^{e_2} \cdots x_k^{e_k}$ is $\dfrac{n!}{e_1!e_2!\cdots e_k!}$, where $\sum_{i=1}^{k} e^i = n$.

Other series:

$$1+2+3+\cdots n-1+n = \frac{n(n+1)}{2}$$

$$1+2^2+3^2+\cdots(n-1)^2+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1+2^3+3^3+\cdots(n-1)^3+n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Functions

Even Functions: A function is even if for all x, $f(x)=f(-x)$. This is the same as symmetry about the y-axis.

Odd Functions: A function is odd if for all x, $f(x) = -f(-x)$. This is the same as symmetry about the origin.

Inverse Trig Functions:

Function	Domain	Range
$Sin^{-1}(x)$	$-1 \le x \le 1$	$-\frac{p}{2} \le Sin^{-1}(x) \le \frac{p}{2}$
$Cos^{-1}(x)$	$-1 \le x \le 1$	$0 \le Cos^{-1}(x) \le p$
$Tan^{-1}(x)$	$-\infty \le x \le \infty$	$-\frac{p}{2} \le Tan^{-1}(x) \le \frac{p}{2}$

Logarithmic Functions:

$$\log_b x + \log_b y = \log_b xy \qquad \log_b x - \log_b y = \log_b \frac{x}{y}$$

$$\log_b x^y = y \log_b x \qquad \log_b x = \frac{\log_n x}{\log_n b}$$

Trig Functions: For the general sine or cosine function,

$A \sin B(x + C) + D$, $|A|$ is the amplitude, $\frac{2p}{B}$ is the period, -*C* is the phase shift, and *D* is the vertical shift.

Greatest Integer Function: *[x]* is the greatest integer that does not exceed the real number *x*.

$[x] \leq x < [x] + 1 \qquad x - 1 < [x] \leq x$

$0 \leq x - [x] < 1 \qquad -x - 1 < [-x] \leq -x$

If n is an integer,

$[x + n] = [x] + n \qquad \left[\frac{[x]}{n}\right] = \left[\frac{x}{n}\right]$

$[x] + [y] \leq [x + y] \leq [x] + [y] + 1$

Complex Numbers

Complex numbers $x + yi$ can be written in the form $r(\cos q + i \sin q)$, where $r = \sqrt{x^2 + y^2}$ and $q = \tan^{-1}\left(\frac{y}{x}\right)$.

Multiplication and Division: For two complex numbers z_1 and z_2:

$$z_1 z_2 = r_1 r_2 [\cos(q_1 + q_2) + i \sin(q_1 + q_2)] \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(q_1 - q_2) + i \sin(q_1 - q_2)]$$

DeMoivre's Theorem: $z^n = [r(\cos q + i \sin q)]^n = r^n(\cos nq + i \sin nq)$

This can be used to find the roots of complex numbers by making *n* the appropriate fraction.

The *n*th roots of 1 when graphed in the complex plane form the *n* vertices of a regular polygon with *n* sides, inscribed the circle with $r = 1$.

Matrices

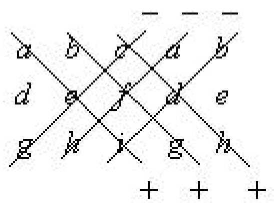
For a 2 by 2 matrix, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

Determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Inverse: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

For a 3 by 3 matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$,

Determinant:



$= aei + bfg + cdh - bdi - afh - ceg$

Inverses of all n by n matrices can be found using row reduction. Write the identity matrix to the right of the original matrix, and multiply and/or add rows to obtain the identity matrix on the left side.

$$\left[\begin{array}{ccc|ccc} a & b & c & 1 & 0 & 0 \\ d & e & f & 0 & 1 & 0 \\ g & h & i & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & a' & b' & c' \\ 0 & 1 & 0 & d' & e' & f' \\ 0 & 0 & 1 & g' & h' & i' \end{array}\right]$$